PART III: MODELING OPERATIONAL RISK

In the previous articles we have given an introduction to the meaning of the term operational risk, explained how it is defined under Basel II and showed a method of structuring the operational risks within a company. In this part we will make a case study of the Standard Approach and an internal model under Solvency II.

INTRODUCTION

Solvency II is scheduled to come into full effect in 2016 for insurers, and with it follows new regulatory requirements. A new feature is that insurance companies must now assign capital to cover the exposure to operational risk. The calculation of Solvency Capital Requirement (SCR) is divided into modules, and with Solvency II, operational risk now constitutes one of the modules.

Solvency II provides a variety of methods to calculate the SCR, which gives companies the freedom to decide on a method that reflects the nature, scale and complexity of their business. When introduced, Solvency II adapted methodologies and definitions similar to those employed by banks in Basel II. We will in this article show how to calculate the operational risk capital requirement, SCR_{op}, both with the Standard Formula introduced by Solvency II, and with an internal model based on ideas from Basel II.

The idea of modeling and showing an example from a Solvency II perspective rather than Basel II, is to illustrate that operational risk is a comprehensive issue that not only concern banks, but also insurers, and should be something that other industries take into account when forming company strategies. The methods employed in this article can in a similar manner be utilized to model the operational risk capital requirement for a bank under Basel II.

SOLVENCY II STANDARD FORMULA

The Standard Formula is a simplified approach, where the capital requirement is calculated as a percentage of premiums. In short, the Solvency Capital Requirement (SCR) is determined adding the contributions from the Basic Solvency Capital Requirement (BSCR), the capital requirement for operational risk (SCR_{op}) and an adjustment for the risk absorbing effect of technical provisions and deferred taxes (Adj) (EIOPA, 2014):

$$SCR = BSCR + SCR_{op} + Adj$$

The BSCR combines capital requirements for six major risk categories in an insurance company, and these are the capital requirements for (EIOPA, 2014):

- Market Risk
- Counterparty Default Risk
- Life Underwriting Risk
- Non-life Underwriting Risk
- Health Underwriting Risk
- Intangible Assets Risk

We will not further explain the calculation of the BSCR, but be content with knowing that it is of interest for the calculation of SCR_{op}. The capital requirement for operational risk is defined as follows (EIOPA, 2014):

$$SCR_{op} = min(0.3 \times BSCR; Op) + 0.25 \times Exp_{ul} \tag{1}$$

where Op is the basic operational risk charge for all business other than life insurance where the investment risk is borne by the policyholders, and Exp_{ul} is the amount of expenses incurred during the previous 12 months in respect of life insurance where the investment risk is borne by the policyholders, excluding acquisition expenses.

SOLVENCY II INTERNAL MODEL

Companies are granted permission to develop their own internal model, provided that it captures the risk profile of the insurance company. A common approach when constructing an internal model is the Loss Distribution Approach (Basel Committee on Banking Supervision, 2011).

LOSS DISTRIBUTION APPROACH

A loss distribution approach (LDA) for modeling operational risk relies on internal loss data, and uses statistical distributions that model the *frequency* of loss events as well as their *severity*.

Mathematically speaking, the LDA originates if the number N of internal operational losses and the severity X of a single loss are assumed to be independent random variables. The loss distribution will then be modeled as follows:

$$L = X_1 + X_2 + \cdots + X_N = \sum_{j=1}^{N} X_j$$

where X_j is the random variable representing the amount of the j^{th} loss, and L is the sum of all losses.

To attain the capital requirement for operational risk, the determination of the probability distribution for the frequency of loss events and the conditional probability of the severity of a loss event, given an operational loss event, are necessary. Following the determination of the probability distributions, Monte Carlo simulations can be run and the 99.5% quantile of the loss distribution calculated. The determination of these components will now be given in a stepwise procedure.

STEP 1: MODELING FREQUENCY

The first step is to determine the number of loss events per year, this will be the frequency. The most common distribution for modeling frequency of operational losses is the Poisson distribution (Basel Committee on Banking Supervision, 2009). The Poisson distribution is a discrete probability distribution, and takes only one parameter (λ), which is the average number of operational loss events in a year. It is a suitable choice since the number of loss events are integer numbers, and we have a system with a large number of possible events that are all rare.

The Poisson distribution is given by:

$$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where

λ is the average number of operational loss events in a year

$k = 0, 1, 2, \ldots$

$k! = k \times k - 1 \times k - 2 \times \cdots \times 1$ is k factorial

$e = 2.71828\ldots$

The mean, λ, can be found by simply observing the number of operational losses per year in a loss database, and calculating the average over all years.

STEP 2: MODELING SEVERITY

While there is a general preference for using the Poisson distribution when modeling the frequency, the severity is somewhat more difficult to model and offers a variety of distributions. The severity is the size of an operational loss event, given that an event has already occurred. The severity need not be an integer like the frequency, but does instead belong to the positive real numbers $\mathbb{R}^+$.

Take as an example that an employee buys two stocks instead of one, then the potential loss could number \$10.25 if that is what a stock is worth. If, however, an acquisition of a company turns out bad, we could have losses numbering millions. As can be readily seen, the severity is a continuous random variable that can vary from very small numbers to extremely large ones.

When selecting an appropriate severity distribution, there are a few different aspects that should be taken into mind. These were proposed by Dutta and Perry (2007):

- Good Fit - Statistically, how well does the method fit the data?
- Realistic - If a method fits well in a statistical sense, does it generate a loss distribution with a realistic capital estimate?
- Well-Specified - Are the characteristics of the fitted data similar to the loss data and logically consistent?
- Flexible - How well is the method able to reasonably accommodate a wide variety of empirical loss data?
- Simple - Is the method easy to apply in practice, and is it easy to generate random numbers for the purposes of loss simulation?

Whatever method one decides to go with, they will all carry fat tails, which means that there are very large events that are probable to occur in the distribution. It is precisely these events we want to hold capital against. The least complex method for modeling severity is using the log-normal distribution, and it is the approach we will utilize.

The probability function of a log-normal distribution is given by:

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(lnx-\mu)^2}{2\sigma^2}}, \qquad x > 0$$

where

μ is the location parameter

σ is the scale parameter

$x \in \mathbb{R}^+ (x > 0)$

STEP 3: MONTE CARLO SIMULATIONS

We are now in a position to calculate the capital requirement, but in order to ensure that we with certainty possess sufficient capital against a potential loss, Monte Carlo simulations must be performed. A random number N is drawn from the Poisson distribution, which will correspond to the number of losses for year one. Then, N numbers are drawn from the log-normal severity distribution. These are summed up to yield the total loss for year one. The process can be repeated, say a million times, and the total loss for each year stored in a cell. When all cells have been calculated, the total loss distribution can be formed.

According to EIOPA (2013), the capital requirement should be subject to a confidence level of 99.5% over a one-year period. The quantile represents, with 99.5% certainty, the maximum loss that will be expected in a single year, and is the amount of capital that must be assessed. Having obtained a total loss distribution, finding the 99.5% quantile is simply a task of finding the 99.5% largest value of all the simulated losses.

Let the loss distribution be called L, and let it be based on simulations of N years, such that $L = [L_1, L_2, \ldots, L_{N-1}, L_N]$. Order the elements of L in ascending order, and find the element at position

$$j = N \times \frac{99.5}{100}$$

Then, the capital requirement for operational risk is the value $L(j)$.

RESULTS

The capital requirement for operational risk can be obtained with data from loss databases, as has been seen in the preceding sections of this article. In the absence of loss data we will in this section make a case study of the capital requirement, with the ambition of comparing an estimated capital requirement with the SCR_{op}.

For the purpose of this study, a fictional local insurance company in Sweden will serve as an example. Reading their annual report for 2013, one can find the number of employees and the cost of personnel for the two year period 2012-2013. The numbers are presented in Table 1:

Year	2013	2012
Number of Employees	391	452
Cost of Personnel	405 618 kSEK	401 356 kSEK

Table 1: Annual report 2013 of our insurance company.

If one assumes that there are 46 working weeks in a year, which is 230 working days, the cost per day of an employee averaged over a two year period is approximately 4200 SEK.

We now make the brave assumption that for every 100 employees, seven small operational losses occur every week in the company while 0.25 huge operational losses occur. Take a small loss to be the cost of an employee for one day, and a huge loss to be the cost of an employee for five days. These are just assumptions that will help us approximate the operational risk capital requirement.

Denote the operational loss per 100 employees and year as L_{op}. It is then given by:

$$
\begin{aligned}
L_{op} &= (\text{\#Small losses per week}) \times (\text{\#Working weeks}) \times (\text{Cost of a small loss}) \\
&\quad + (\text{\#Huge losses per week}) \times (\text{\#Working weeks}) \times (\text{Cost of a huge loss})
\end{aligned}
$$

The average number of employees over the two year period 2012-2013 is 421.5, so by multiplying L_{op} by 4.215, we arrive at the approximated operational loss per year, L_{tot}:

$$
\begin{aligned}
L_{tot} &= 4.215 \times ((7 \times 46 \times 4200) + (0.25 \times 46 \times 5 \times 4200)) \, \text{SEK} \\
&= 6.7 \, \text{MSEK}
\end{aligned}
$$

As a minimum, the insurance company should, in the light of our simplified approach, hold at least 6.7 MSEK in capital against operational losses. How does this compare to the SCR_{op}?

The SCR_{op} can be calculated using the formula provided in (1) in the section "Solvency II Standard Formula". In the annual report of our fictional local insurance company the variable $TP_{non-life}$ can be found, and it is the only variable we will be needing to calculate the SCR_{op}. With $TP_{non-life} \approx 2500$ MSEK the SCR_{op} is given by:

$$
\begin{aligned}
SCR_{op} &= 0.03 \times TP_{non-life} \\
&= 0.03 \times 2500 \, \text{MSEK} \\
&= 75 \, \text{MSEK}
\end{aligned}
$$

As can be readily seen, the SCR_{op} is more than ten times as large as the estimated operational loss per year. What conclusions can we draw from this?

DISCUSSION

With our simplified approach to estimating the yearly operational loss, we saw that it was merely a tenth of the operational risk capital requirement, SCR_{op}. To justify the claims in our case-based reasoning, consider when the approximation equals the SCR_{op}, i.e., when $L_{tot} = SCR_{op}$. It would require as much as 87 small losses and one huge loss per week, or seven small losses and 17 huge losses per week, as compared to the seven small and 0.25 huge operational losses we estimated. For a company of around 400 employees, it would be rather concerning if they experienced as much as 87 small operational losses per week.

Seeing that the approximation can be argued for, we identify some interesting consequences. Investing in an internal model could prove to be a sustainable solution for the company, as it would yield results similar to our estimation, allowing to free up capital. It is, however, something that has to be evaluated individually for every unique case.

What has been seen in previous sections, is that the internal model is capable of capturing the risk profile of a company, yielding a sophisticated risk management. On the contrary, following the implementation of an internal model are continous supervisory requirements. They can be both costly and time-consuming. A possibility is for a company to adapt an internal model within the company, but externally holding capital with the Standard Formula. This gives insight into the real situation of the company, and saves both time and money meeting supervisory requirements. Regardless of the preferred method for calculating the operational risk capital requirement, operational risk is something that should be dealt with in a cautious and regarding way.

SUMMARY

In this article we have calculated the operational risk capital requirement through a case-based approach and compared the capital requirement with the Solvency II Standard Formula. We have explained how the Loss Distribution Approach (LDA) can be utilized to model the operational risk by using loss databases, and shown in a stepwise procedure how one can predict the maximum loss that will occur in a single year.

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CONTACT US

Do you want to know more about Kidbrooke Advisory?
Please contact us for more information.

e. info@kidbrooke-advisory.com

a. Kidbrooke Advisory Aktiebolag
Birger Jarlsgatan 20
SE-114 34 Stockholm
Sweden

FREDRIK DAVÉUS
Managing Partner
+46(0)761 170 009
fredrik.daveus@kidbrooke.com

EDVARD SJÖGREN
Partner
+46(0)733 182 583
edvard.sjogren@kidbrooke.com

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