PORTFOLIO CONSTRUCTION PART II: SAMPLING & OPTIMIZATION

The main findings of the previous article in the series, Kidbrooke Advisory (2019a), reveal that model- and, in particular, parameter uncertainty has a significant impact on the optimal portfolio allocations. The second part of the "Portfolio Construction"-series explores whether introducing parameter uncertainty to the model would improve the out-of-sample performance of the optimal portfolio. In addition, two adjustments to regular utility optimization are proposed and tested.

OUT-OF-SAMPLE PERFORMANCE

Throughout this article series, our aim is to find the combination of optimization and simulation methods that on average provide the best out-of-sample performance. Since the real world only consists of a single trajectory with a limited amount of data, it is difficult to use it to produce any significant results. Therefore, it is assumed that the model of the real world is known to us. That way, a simulated world with a lot of data can be used to evaluate a portfolio's out-of-sample performance. The simplest way of doing so is by using the algorithm, denoted **Static out-of-sample** performance, described by the following steps:

1. Simulate a large number N of scenarios using the assumed model of the real world. Denote this set $S_{real-world}$.
2. For every asset, use scenario i from N simulated real world scenarios for model calibration.
3. Using the calibrated model from step 2, simulate a large number M of scenarios and use these to optimize your portfolio.
4. Repeat step 2 and 3 using scenario $i = 1, .., K$ from the real world scenario set, $S_{real-world}$. For every scenario save the resulting optimal portfolio weights.

5. Calculate the performance for each and one of the K optimal portfolios on the original data set, $S_{real-world}$. The average out of sample performance is calculated as the average of the K out of sample performances.

The portfolio performance is measured by its expected utility, represented by a constant relative risk aversion utility function. There are many advantages of using utility-based optimization instead of e.g. Markovitz's mean-variance optimization; for a brief review of the utility theory and a full motivation of our choice of performance measures, see Kidbrooke Advisory (2019b).

Our previous article shows that the historical model behaviour does not necessarily represent the future. An additional evaluation, where calibration and evaluation are performed on separate time periods, will therefore be included in the analysis. This measure, which is denoted **Predictive out-of-sample** performance, would with a few modifications work in the same way as the **Static out-of-sample**. Firstly, the real world model used to simulate $S_{real-world}$ will only be calibrated to the first half of the historical observations. The second half of the observations will be used to calibrate and simulate a new set of scenarios, $S_{future-world}$. To measure **Predictive out-of-sample** performance, simply replace $S_{real-world}$ by $S_{future-world}$ in step 5 of the algorithm described above.

Once again, our asset universe contains an AAA Swedish corporate bond and an Emerging Market equity index. However, for this evaluation, the Global equity index has been replaced by a Canadian equity index, since the former dominated the Emerging Market equity index in the optimal portfolios.

BAYESIAN SAMPLING

The previous article in the series suggest that a basic multivariate normal distribution is a sufficient simulation model. This is based on the significant parameter uncertainty, which overshadows the model sensitivity. Using a Bayesian sampling method is one way to introduce parameter uncertainty to our model. By changing the prior, the Bayesian approach could also be used to capture the uncertainty in future model behaviour. However, to limit the scope of this article, our prior is chosen to capture only the calibration uncertainty of the parameters.

Assume that there is a real world model that new scenarios are simulated from. If a standard frequentistic approach is used, the model parameters would be set to the parameters that would have generated the historical observation with the highest probability. However, this ignores any uncertainty of the model parameters. When using Bayesian sampling, the parameters of the model are assumed to be stochastic rather than fixed. Using a multivariate normal model, this implies that the drift μ and covariance Σ are drawn from appropriate distributions before every set of multivariate normal variables is simulated. For a more extensive account on the Bayesian sampling method in the multivariate normal case, see e.g. Bernardo & A.F.M. (2000).

OPTIMIZATION METHODS

When the proposed Bayesian sampling method is used, the simulated scenarios display a more diverse distributional behaviour in comparison to basic multivariate normal model. Since all parameters are still centred around the same points, however, it is reasonable to believe that an average behaviour of the model would not be affected in a significant way. The utilization of the diverse model behaviour is achieved by introducing two adjustments to the regular utility optimization.

The optimization procedures proposed below are created to solve some of the known problems of model based portfolio optimization. The fact that the resulting optimal portfolios perform well on average lacks theoretical underpinning, and therefore they will be assessed solely based on their out-of-sample performance.

AVERAGE BAYESIAN OPTIMIZATION

The idea behind the **Average Bayesian optimization** approach is to reduce the probability of extreme weights. This is achieved by calculating a number of optimal portfolios for different sets of model parameters, and setting the optimal portfolio as the average of all these optimal portfolios.

1. Sample m sets of multivariate normal model parameters.
2. For every set of parameters, simulate n trajectories.
3. For every simulation set, find the initial optimal portfolio weights by maximizing the utility.
4. Set the optimal portfolio to the average of all m optimal portfolio weights.

MAX-MIN BAYESIAN OPTIMIZATION

Instead of finding a portfolio that performs as well as possible on average, the **Max-Min Bayesian optimization** approach finds the portfolio that performs as well as possible in the worst case given a lot of different model parameter sets. This would create a portfolio that is robust to a changed model behaviour.

1. Sample m sets of multivariate normal model parameters.
2. For every set of parameters simulate n trajectories.
3. Find the portfolio that has the highest expected utility over the simulation set i with the lowest expected utility.

The observant reader will notice that as m grows, the risk of the worst simulation set will increase, resulting in an optimal portfolio that converges to investing everything in the asset with the lowest risk. To make the results robust to an increasing number of simulation sets, one could maximize the utility for a specific percentile of expected utilities, rather the worst expected utility. This also makes it possible to adjust the level of risk considered for the method, to further suit risk preferences.

RESULTS & CONCLUSION

Recalling the main findings of the first part of the "Portfolio Construction"-series, model- and, in particular, parameter uncertainty has a significant impact on the optimal portfolio. Therefore, the Bayesian sampling method was proposed to introduce some parameter uncertainty to the model. In this section, the results from the Bayesian simulation model are presented and compared to those obtained through using a regular multivariate normal model. Optimal portfolios are created using the **Average Bayesian-** and the **Max-Min Bayesian** optimization frameworks, along with a regular utility optimization approach. The out of sample performance is evaluated using both the **Predictive out-of-sample-** and the **Static out-of-sample-** measures. Since the results using both out-of-sample performance measures pointed towards very similar results only the **Predictive out-of-sample** results will be displayed.

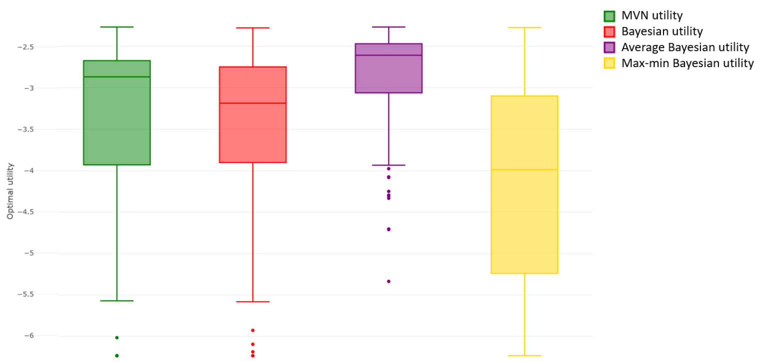

Figure 1: ***Predictive out-of-sample*** *utility for regular utility optimization using a multivariate normal distribution (MVN Utility) with Bayesian sampling (Bayesian utility), the* ***Average Bayesian-*** *and* ***Max-Min Bayesian*** *optimization method. For comparison the utility of an equal weight portfolio is included.*

Figure 1 shows the distribution of **Predictive out-of-sample** utilities for the considered optimization methods. As implied by the plot, Bayesian sampling alone did not improve the optimal portfolio utility. When using Bayesian sampling along with an Average utility optimizer, however, a considerable improvement was distinguishable; the **Average Bayesian** optimization method on average provided the highest out-of-sample utility. The **Max-Min Bayesian** optimization method, on the other hand, performed rather poorly in comparison. What is even more surprising is that the worst out-of-sample performance of the **Max-Min Bayesian** is so bad, given that this was the statistic of interest when performing the optimization.

Remember that the considered set of assets consisted of merely three assets. It should also be noted that the actual optimal portfolio over the evaluation set was close to being equally weighted. This would of course favour a strategy like the **Average Bayesian** method which by construction creates balanced weights. Since the **Max-Min Bayesian** method optimizes with respect to the worst case, the asset with the lowest risk is bound to get a larger portfolio weight, leading to a less balanced portfolio and bad out-of-sample performance.

If a larger and more realistic set of assets would have been considered, it is plausible that the outcome would have been different. In the third and concluding part of the “Portfolio Construction”-series the focus will be on constructing optimal portfolios using real data from 170 funds. Two new optimization methods will be introduced, and a solution to the problem of optimizing weights for a large number of assets will be proposed.

Bibliography

Bernardo, J.M., & A.F.M., Smith. 2000. *Bayesian Theory*. Wiley, Chichester.

Kidbrooke Advisory. 2019a. *Portfolio Construction Part I: Parameter & Model Uncertainty*.

Kidbrooke Advisory. 2019b. *Utility vs Mean Variance Optimization*.

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